Vision Empower & XRCVC

Teacher Instruction KIT

EXPONENTS AND POWERS

Syllabus: Karnataka State Board
Subject: Mathematics
Grade: 7th
Textbook Name: MATHEMATICS – Text cum Workbook (Revised) – Seventh standard
Chapter Number & Name: 13 Exponents and Powers

1. OVERVIEW

1.1 OBJECTIVE & PREREQUISITES

Objective

Students will be able to:

- Define the term exponent.
- Define and interpret the meaning of a^n, where n is a positive integer.
- Demonstrate the ability to use the properties of exponents.

Prerequisite Concept

- Multiplication

TIK_MATH_G5_CH11_Multipication

Content Index

*Kindly Note: Activities marked with * are mandatory*

2. LEARN

2.1 KEY POINTS

Exponents Rules/ Laws of Exponents

- Multiplication/ Product Rule: When multiplying two exponents that have the same base, add the exponents. For any non-integer, where m and n are whole numbers,

  $a^m * a^n = a^{m-n}$

- Division-Quotient Rule: When dividing two exponents that have the same base, subtract the exponents. For any non-integer a, $a^m / a^n = a^{m-n}$, where m and n are whole numbers and $m > n$.

- Zero Rule: Any non zero number raised to the power of zero equals 1. $a^0 = 1$.
- Power of a Power Rule: If 'a' is any non-zero integer, and 'm' and 'n' are whole numbers then, $(a^m)^n = a^{mn}$.
- An exponent refers to the number of times a number is multiplied by itself.
- Expanded form is a way of writing numbers to see the math value of individual digits. For example, $1000 = 10 * 10 * 10$ or $123 = 100 + 20 + 3$.

2.2 LEARN MORE

3. ENGAGE

3.1 INTEREST GENERATION ACTIVITY

Activity 1: Large numbers

Materials Required: None

Prerequisites: None

Activity Flow

The below activity helps the students to understand the need for exponents and powers in real life.

- Discuss the following questions with the students?
  - *a.* What is the distance between your school and your house?
  - *b.* What is the distance between Bangalore and Chennai?
  - *c.* What is the distance between India and the United States of America? Note: Give the idea of the distance between two places in kilometers and discuss how the number gets bigger.
  - *d.* Give the names of five nearest planets to the sun?
  - *e.* What is the distance between the Sun and Mercury?

f. What is the distance between the Sun and the Earth?

- Inform the approximate distances from the sun to its nearest planets.
  - *a.* MERCURY 60000000 km
  - *b.* VENUS 100000000 km
  - *c.* EARTH 150000000 km
  - *d.* MARS 200000000 km
- Ask the students to read the distance between the Earth and the sun.
- Explain to the students that scientists engage in space research and work with extremely large numbers. Those are difficult to read as it is.
  For example, Pluto is approximately 6000000000 km away from the sun.
  The star Cygni is approximately 300000000000000 km distance from the Earth.
- Explain to the students, the scientists have a shorter way to represent the larger or very smaller number. Exponents and powers are used to represent the larger and smaller numbers.

3.2 CONCEPT INTRODUCTION ACTIVITIES

Exponents

Activity 1: Exponents

Materials Required: Braille cards of 10's, 100's and 1000's and marbles

Prerequisites: Multiplication

Activity Flow

Exponent is the number of times a number is multiplied by itself. Exponents are written as x^y; where x is the number and y is the exponent or power. Example $2*2*2*2*2=2^5$

A larger number can be written in a shorter way using exponents.

- Give the following examples to the students.

1. $100=10^2$.
2. $1000=10*10*10=10^3$.
3. $10000=10*10*10*10=10^4$.

- Explain to the students that the shorter way to represent 10000 is 10^4 . In 10^4 , 4 is the exponent which means the number 10 is multiplied 4 times. This number 10 can be read as 10 raised to the power of 4 or 10 to the power 4. 10^4 is the exponent form of
  10000. 10 is the base and 4 is the exponent.

To teach this,

- Separate students into small groups.
- Distribute the "Braille cards" and the marbles to each group.
- Ask the students to write the following numbers in multiples of 10 using the "Braille cards" in row-wise.
    a. 100
    b. 1000
    c. 10000
    d. 100000

For example, placing two 10's for the number 100.

- Ask the students how many 10's are there for the number 100. Then, ask them to take that many marbles in their hands. Tell them that marbles are going to represent the exponents and cards are going to represent the base.

  $100 = 10 * 10$.

  ○ 10 is multiplied two times to get 100 so, 2 is the exponent and 10 is the base.

  ○ To write its exponential form, ask the students to place one 10's card then 2 marbles on top of the card.

  ○ Ask them to read the above representation as 10 raised to the power of 2. Explain 10^2 means 10 is multiplied two times.

  ○ Ask the number which is on the card and explain that is the base. And the number of marbles on the top of the card is the exponent.
- Emphasizing that exponent is the number indicates how many times the number multiplied itself.
- Ask the students to write 1000, 10000, 100000 in multiples of 10. Then ask them to represent it in exponent form using the given materials.
- Clarify the doubts and discuss the need for exponents.

Express numbers in exponential form

Activity 2: Express numbers in exponential form

Materials Required: None

Prerequisites: Multiplication

Activity Flow :

- Ask the exponents and base of the following numbers.
    a. 2^6
    b. 4^4

c. 11^2

d. 10^8

- Ask the students to express the following in the exponential form:
  a. 6 *6 *6 *6
  b. 5*5*7*7*7*7
  c. 8*8
- Ask the students to find the value of:
  a. 2^2
  b. 2^4
  c. 10^2
  d. 5^3
- Clarify the doubts of the students.

Power of one

Activity 3: Power of one

Materials Required: None

Prerequisites: Multiplication

Activity Flow :

Any number raised to the power of one equals the number itself.

$a^1 = a$

- Ask the students to find the value of
  a. 2^1
  b. 3^1
  c. 4^1
- Ask the students to identify a pattern from the above answers.
  For example,
  $2^1 = 2$;
  $3^1 = 3$;
  $4^1 = 4$;
- Explain that any number raised to the power of one equals the number itself.
  For example $2^1 = 2$; 1 is the exponent; 2 is the base; 2^1 means the number 2 is multiplied one time so the value is 2.

Multiplying and dividing powers with the same base

Activity 4: Multiplying powers with the same base
Materials Required: Braille cards of X and Marbles
Prerequisites: Multiplication and Addition

Activity Flow
When multiplying two exponents that have the same base, add the exponents. For any non-integer, where m and n are whole numbers, $a^m a^n = a^{m+n}$.

For example, $2^3 * 2^2 = 2^{3+2} = 2^5$.

- Group the students into pairs
- Distribute the " Braille cards of x or any variables and marbles.
- Ask the students to write x raised to the power of 4 and x raised to the power of 3 using the cards and marbles.
  For example, placing 4 marbles on top of the card X to represent X raised to the power of 4. Similarly, ask the students to write X raised to the power of 3. i.e X^4 and X^3
- Ask the students to read the expression and to expand the exponents using the X cards.

$x^4 * x^3 = x * x * x * x * x * x * x$

- Explain to the students that we have a product rule to multiply the exponents with the same base.
  For example, $X^4 * X^3$, the exponents have the same base, so add the exponents directly.
  I.e $x^4 * x^3 = x^{4+3} = x^7$
  Instead of expanding the terms, use the exponents' law to simplify.
  Consider $X^{34} \times X^{34}$, here expansion is a difficult and time-consuming process.
  Directly apply the product rule to find the answer.

- Discuss and clear the doubts of the students.

Note: The multiplication rule applies just to the product, not to the sum of two numbers.
For example, $2^2 + 2^3 \neq 2^{2-3}$.

Activity 4: Dividing powers with the same base using quotient rule of exponents.
Materials Required: Marbles and "Braille cards.
Prerequisites: Subtraction

When dividing two exponents that have the same base, subtract the exponents.
Quotient rule/ Division Rule: For any non-integer a, $a_m a_n = a_{m-n}$, where m and n are whole numbers and m > n.

- Consider $3^4 \div 3^2$
- Ask the students to expand the exponent.
  i.e. $(3\times3\times3\times3)/3\times3$.
- Then ask the students to simplify: $(3\times3\times3\times3)/3\times3$
  $(3\times3\times3\times3)/3\times3 = 3^2$
- Now, explain the Quotient rule of exponents to the students with an example.
  Consider $3^4 \div 3^2$, both exponents have the same base.
  $3^4 \div 3^2 = 3^{4-2}$ (applying quotient rule, I.e., subtracting the exponents).
  $3^4 \div 3^2 = 3^{4-2} = 3^2$
- Explain to the students that instead of expanding and simplifying the exponents, directly apply the quotient rule wherever it's applicable.
- Give the following questions to simplify. The questions are:
  a. $2^9 \div 2^3$
  b. $10^8 \div 10^4$
  c. $7^{13} \div 7^{10}$
  d. $20^{11} \div 20^8$

Power of a power

Activity 5: Power of a power

Materials Required: Braille cards of exponents and marbles
Prerequisites: Multiplication and addition

Activity Flow

- Group the students into pairs.
- Distribute the materials to each pair and ask the students to take one exponent card from the given materials.
- Ask the base and the exponent of the expression.
- Then ask them to place 4 marbles on top of the exponent card. Again ask the base and the exponent of the expression.

  For example, $(3^2)^4$, 3^2 is the exponent card and 4 marbles on top of it. Here, the base is 3^2 and the exponent is 4.
- Ask the students to read and to explain their expressions. Give some time to the students to think and to discuss with their partners.

- Explain, power of the power to the students.

  Consider $(3^2)^4$

  $(3^2)^4$ means multiplying 3^2 four times.

  The above expression can be written as:

  $(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

- Give the practice to write the exponents with power.

Activity 6: Power of power using the power rule
Materials Required: None
Prerequisites: Multiplication

Activity Flow
Power Rule: To raise a power to a power, multiply the exponents. For any non-zero integer 'a', where m and n are whole numbers, $\left(a^m\right)^n = a^{mn}$

- Explain the power rule to the students with an example and compare it with the previous activity.
  $\left(2^2\right)^3 = 2^{2\times 3} = 2^6$
- Ask the students to simplify the following expressions using the power rule.
  a. $\left(2^3\right)^3$
  b. $\left(3^2\right)^2$
  c. $\left(a^3\right)^3$
  d. $\left(a^m\right)^3$
- Discuss the use of the power rule and clarify their doubts.

Multiplying powers with the same exponents

Activity 7: Multiplying powers with the same exponents.
Materials Required: None
Prerequisites: Multiplication

Activity Flow
Power of a product rule: If 'a' is any non-zero integer, and 'm' is a whole number then,

$a^m \times b^m = (ab)^m$

For example,

$2^3 \times 3^3 = 6^3$

Consider

$2^3 \times 3^3$

- Explain to the students that in both terms the exponents are the same. If the exponents are the same in multiplication then apply the power of a product rule. I.e. multiplying the base and writing the exponent as it is.

  I.e., $2^3 \times 3^3 = (2 \times 3)^3 = 6^3$
- Explain the above example using normal multiplication.

$$2^3 \times 3^3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= 6 \times 6 \times 6 = 6^3$$

(or)

- Ask the students to find the answer without using the power of a product rule. Then ask them to compare with the power of a product rule. We can simplify the exponents using the law of exponents.

$2^3 \times 3^3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 6 \times 6 \times 6$

$= 6^3$

- Ask the students to simplify the following expressions using the power of a product rule.

  a. $(2 \times 3)^5$

  b. $(X \times Y)^3$
- Discuss and clarify their doubts.

Dividing powers with the same exponents

Activity 8: Dividing powers with the same exponents.

Materials Required: None

Prerequisites: Multiplication

Activity Flow

If a and b are any non-zero integers and m is a whole number then, $a^m \div b^m = (a/b)^m$

For example

$2^2 \div 3^2 = (2/3)^2$

Similarly,

$$(2^2 \times 3^2) = (2 \times 3)^2$$

- Explain how to simplify the following exponents to the students.
  Consider
  $2^2 \times 3^2$

  It can be written as $(2 \times 2)/(3 \times 3) = 2/3 \times 2/3 = (2/3)^2$

  (or)

  $(2 \times 2) \div (3 \times 3) = 4/9 = (2/3)^2$

  4 can be written as 2^2 and 9 can be written as 3^2
- Explain the above law to the students using the given example that, if the terms have the same exponent, it can be written as a common exponent to both the numbers.
  therefore,
  $2^2 \div 3^2 = (2/3)^2$
- Ask the students to simplify the following questions:
  a. $(3/5)^4$
  b. $(-4/3)^2$
  c. $(3 \times 2)^2$

Power with exponent zero

Activity 9: Power with exponent zero

Materials Required: None

Prerequisites: Multiplication

Activity Flow

If 'a' is a non-zero integer or a non-zero rational number then, $a^0 = 1$

- Ask the students to find the value of the following exponents.
  a. 2^5
  b. 2^4
  c. 2^3
  d. 2^2
  e. 2^1
  f. 2^0

Note: Ask the students to find the pattern of the above numbers to guess the value of 2^0

For example $2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^3 = 2 \times 2 \times 2 = 8$

$2^2 = 2 \times 2 = 4$

$2^1 = 2$

$2^0 = ?$

The pattern is

$16 / 2 = 8$, $8 / 2 = 4$, $4 / 2 = 2$, $2 / 2 = 1$

To get the next number, divide the number by 2 so, $2 / 2 = 1$

The value of $2^0 = 1$

- Explain to the students that anything to the power zero is equal to 1.

Expressing large numbers in the standard form

Activity 10: Expressing large numbers in the standard form

Materials Required: None

Prerequisites: Decimals, Refer to TIK_MATH_G7_CH2_ Decimals (activity 1,2,3).

Activity Flow

In the beginning, students had a discussion about large numbers.

For example, the distance of Pluto from the sun is approximately 6000000000 . It is difficult to read as well as to write. There is a shorter way to write this number. It is called a standard form.

Consider 6000

It can be written like 6×1000

1000 can be written like

$10 \times 10 \times 10 = 10^3$

6 can be written as 6.0 (in decimal form)

Therefore, $6.0 \times 1000 = 6.0 \times 10^3$

Ask the students to write the following numbers in its standard form:

a. 1000

b. 40000

c. 300000

Another way to explain the standard form.

Consider 345000

- In the above example 345000 , there is no decimal point, so, assume the decimal point to be at the right end. From there, shift the decimal to the front of the first

digit. For this number 345000, shift the decimal to 5 digits towards the left. To move the decimal point 5 digits towards the left, multiply the number by 10^5

Therefore,

$$345000 = 3.4510^5$$

3.3 LET'S DISCUSS: RELATE TO DAILY LIFE*

Exponents are used in a variety of ways. Most often, exponents are used to representing a very big or very small number. Exponents are used to describing the memory of the computer. For example, one gigabyte of RAM is equal to 1×10^9 bytes. To measure the strength of Earthquakes will use exponents. For example, level 1 earthquake means 1×10^1; level 2 means 1×10^2, etc. Exponents are used to counting the things which grow very quickly.

Ask the students to think of an example, which grows very quickly. For example, bacteria, the virus which multiplies very fast.

4. EXERCISES & REINFORCEMENT

4.1 PRACTICE EXERCISES

Practice and Recall

Activity 1: RECALL AND PRACTICE

Materials Required: None

Prerequisites: Multiplication, Addition, and Subtraction

Activity Flow

1) Find the value of:
   - a) 2^2
   - b) 9^3
   - c) 11^2
   - d) 5^4
2) Express the following in exponential form:

a) $6\times6\times6\times6$

b) $t\times t\times t\times b\times b\times b$

c) $5\times5\times7\times7\times7$

d) $2\times2\times a\times a$

e) $a\times a\times a\times c\times c\times c\times c\times d$

3) Express each of the following numbers using the exponential notation:
   - a) 512

b) 343

c) 729

d) 3125

4) Identify the greater number, wherever possible, in each of the following?

a) 4^3 or 3^4

b) 5^3 or 3^5

c) 2^8 or 8^2

d) 100^2 or 2^{100}

e) 2^{10} or 1^0

5) Express each of the following as product of powers of their prime factors:

a) 648

b) 405

c) 540

d) 3600

6) Simplify:

a) 2×10^3

b) 72×2^2

c) $2^3\times5$

d) 3×4^4

e) 0×10^2

f) $5^2\times3^3$

g) $2^4\times3^2$

$3^2\times10^4$

7) Simplify:

a) $(-4)^3$

b) $(-3)\times(-2)^3$

c) $(-3)^2\times(-5)^2$

d) $(-2)^3\times(-10)$

8) Compare the following numbers:

a) 2.7×10^{12} , 1.5×10^8

b) 4×10^{14} , 3×10

9) Using laws of exponents, simplify and write the answer in exponential form:

a) $3^2\times3^4\times3^8$

b) $6^{15}\div6^{10}$

c) $a^3\times a^2$

d) $7^x \times 7^2$

e) $\left(5^2\right)^3 \div 5^3$

f) $2^5 \times 5^5$

g) $a^4 \times b^4$

h) $\left(3^4\right)^3$

i) $(2^{20} \times 2^{15}) \times 2^3$

j) $8^t \div 8^2$

10) Simplify and express each of the following in the exponential form

a) $(2^2 \times 3^4 \times 4)(3 \times 32)$

b) $((5^2)^3 \times 5^4) \div 5^7$

c) $25^4 \times 5^3$

d) $(37^2 \times 11^8)(21 \times 11^3)$

e) $3^7 \div (3^4 \times 3^3)$

f) $2^0 + 3^0 + 4^0$

g) $2^0 \times 3^0 \times 4^0$

h) $\left(3^0 + 2^0\right) \times 5^0$

i) $(2^8 \times a^5) \div (4^3 \times a^3)$

j) $\left(a^5 / a^3\right) \times a^8$

k) $(4^5 \times a^8 b^3) / (4^5 \times a^5 b^2)$

l) $(2^3 \times 2)^2$

11) Say true or false and justify your answer:

a) $10 \times 10^{11} = 100^{11}$

b) $2^3 > 5^2$

c) $2^3 \times 3^2 = 6^5$

d) $3^0 = (1000)^0$

12) Express each of the following as a product of prime factors only in exponential form:

a) 108×192

b) 270

c) 729×64

d) 768

13) Find the number from each of the following expanded forms:

a) $\left(8 \times 10^4\right) + \left(6 \times 10^3\right) + \left(0 \times 10^2\right) + \left(4 \times 10^1\right) + (5 \times 100)$

b) $\left(4 \times 10^5\right) + \left(5 \times 10^3\right) + \left(3 \times 10^2\right) + \left(2 \times 10^0\right)$

c) $\left(9\times10^{5}\right)+\left(2\times10^{2}\right)+\left(3\times10^{1}\right)$

14) Express the following numbers in standard form:

a) $5,00,00,000$

b) $70,00,000$

c) $3,18,65,00,000$

d) $3,90,878$

e) 39087.8

f) 3908.78

12) Express each of the following as a product of prime factors only in exponential form:

a) 108×192

b) 270

c) 729×64

d) 768

4.2 IMPORTANT GUIDELINES*

Exercise Reading

It is very important that the children practice their learnings as well as their Reading. Hence have the children read out the newly learned concepts from their textbooks or other available resources.

Perform Textbook Activity

It is good practice to have the children perform the textbook activities. Your textbook activities might not be accessible hence go through this resource to learn how to make textbook content accessible

Provide Homework

To evaluate their understanding and to help the student revise and implement the new learnt concept ensure to provide them with homework. Students should perform one or two of the questions mentioned above or from the textbook exercises with the teacher in Class and the remaining may be given for homework. Also, ensure that the student knows their special skills linked to independently using their accessible books as it will be critical to doing homework independently

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